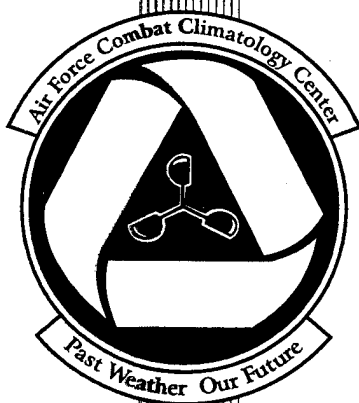


A COMPARISON of AIRCRAFT ICING FORECAST MODELS

by
Capt Daniel Cornell
Capt Christopher A. Donahue
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DECEMBER 1995

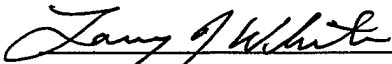
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PREFACE

This report documents the results of AFCCC Project 920241, completed by AFCCC's Simulation and Techniques Branch (AFCCC/SYT). The project analysts were Captains Daniel Cornell, Christopher Donahue, and Chan Keith.

A study was performed to determine which objective method for forecasting icing type and intensity compared best with pilot reports (PIREPs) of aircraft icing. Three operationally run forecast algorithms and discriminant analysis procedures were used to forecast aircraft icing type and intensity from rawinsonde and NGM analysis data in the vicinity of observed occurrences of icing. Overall, the RAOB procedure used at AF Global Weather Central (AFGWC) proved to be the best — agreeing with 67 percent of the PIREPs with regard to type and 42 percent with regard to intensity. Stratification of the PIREP data by aircraft type showed that for jet aircraft these percentages increased to 73 percent (type) and 53 percent (intensity). The difficulty in classifying aircraft icing type and intensity conditions based solely on temperature, moisture, and atmospheric stability were clearly reflected in the results of discriminant analysis procedures, where classification accuracy was no better than that expected by chance. Until a more accurate procedure to forecast the type and intensity of aircraft icing is identified, AFCCC will use AFGWC's RAOB procedure to produce climatologies of aircraft icing type and intensity.

The authors wish to acknowledge the programming expertise provided by TSgt Todd Knauer, SSgt C. Michael Whitney, A1C Douglas Stave, A1C Dave Baldwin and A1C Lance Fraley. We are especially grateful to Paul Schultz of the National Oceanic and Atmospheric Administration's Forecast Systems Laboratory (NOAA/FSL) for providing the PIREP data, and to Dr Richard Jeck of the Federal Aviation Administration (FAA) Technical Center for his advice and encouragement.

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Chapter 1

INTRODUCTION

The ability to assess the frequency with which aircraft will be exposed to the meteorological conditions conducive to aircraft icing is critical for the efficient design and safe operation of all types of aircraft. Currently, AFCCC's climatological support includes providing the frequency of occurrence of icing at user-specified levels. When determined from a large period of record, the *frequency* of occurrence approximates the *probability* of occurrence.

However, customers require that we specify the icing environment in terms of icing type and intensity along with a simple probability. As a result, AFCCC opened a project to examine the various algorithms used to specify icing type and intensity. This report documents the results of a study to compare forecasts of aircraft icing types and intensities from several icing forecast algorithms to actual recorded pilot reports (PIREPs) of aircraft icing. Three of these forecast

techniques are operational algorithms used by the military to forecast aircraft icing. They are:

- Air Force Global Weather Central's (AFGWC) RAOB icing algorithm
- The icing routine used in the Navy's Naval Oceanographic Data Distribution System (NODDS) software
- The icing routine used in the Navy's Tactical Environmental Support System (TESS) software.

A multivariate statistical technique (discriminant analysis) was used to classify the icing observations. An overall comparison (as well as one stratified by aircraft characteristics) were made of the *forecast* type and intensity and those reported by PIREPs. The objective of the study was to identify the best available means to quantify (in terms of type and intensity) aircraft icing conditions from upper-air data.

Chapter 2

LITERATURE REVIEW

2.1 Types of Icing. Generally, three types of icing are considered: clear, rime, and mixed. Clear ice usually occurs at temperatures just below freezing. It is clear or translucent and is the result of the relatively slow freezing of large supercooled liquid water droplets that spread out upon impact. On the other hand, rime icing occurs at cooler temperatures and is the result of the instantaneous freezing of small supercooled droplets as they strike the aircraft. The fact that the droplets maintain their spherical shape results in air spaces between the droplets. Because of these air spaces, rime ice has a milky appearance and a brittle composition. Mixed icing occurs at intermediate temperatures and is a combination of clear and rime icing. In addition to temperature and drop size, other factors determining icing type are liquid water content, air speed, and the size and shape of the airfoil (Air Weather Service, 1980). Hansman (1989) showed that given the same temperature and droplet size, an increase in liquid water content (LWC) can cause a transition from rime to mixed icing. The higher the rate of LWC impaction, the more likely the unfrozen water will flow back along the wing before adequate heat can be released and the supercooled water freeze.

2.2 Icing Intensity. The subcommittee for Aviation Meteorological Services in the Office of the Federal Coordinator for Meteorology (1968) recommends four categories for the reporting of aircraft icing: (1) Trace - when icing becomes perceptible, (2) Light - the rate of accumulation creates a problem for prolonged flight (over one hour), (3) Moderate - the rate of accumulation presents a hazard even for short encounters, and (4) Severe - the rate of accumulation is such that deicing equipment fails to control the hazard; immediate diversion is necessary.

Many factors combine to determine the intensity of aircraft icing, including temperature, drop size, LWC, as well as non-meteorological factors such as airspeed and airfoil configuration. The difficulty in measurement and the variability of these factors with altitude, position, and time, coupled with variable aircraft sensitivity make forecasting and identifying icing conditions difficult. Forecasts of icing intensity should represent the probable maximum intensity based upon meteorological conditions expected to exist at a point in space and time, and not necessarily the intensity the aircraft will encounter due to variations in non-meteorological factors, which influence the actual ice accumulation of a particular aircraft under a given set of meteorological conditions (Air Weather Service, 1980).

Generally speaking, icing occurs only at temperatures between 0 and -40 °C and a higher LWC implies greater ice accretion. For slow moving fixed wing aircraft, most moderate to severe icing occurs at air temperatures between -5 and -13 °C. At warmer temperatures, the aerodynamic heating and latent heat release results in a slushy surface that may blow off the trailing edge of the wing. As air speed increases, additional thermodynamic heating and increased shear forces tend to diminish the ice accretion. Severe ice accretions at high subsonic speeds requires lower temperatures but this is moderated somewhat by lower LWCs (Hansman, 1989). Droplet size, usually categorized in terms of the median volume diameter (MVD), is another major factor influencing icing intensity. Pobanz and Marwitz (1990) document conditions of low liquid water content but hazardous ice accumulation due to large drop sizes. Unfortunately, because so many environmental factors influence the drop size distribution, the

CHAPTER 2

MVD is not easily parameterized. Politovich (1993) states that aircraft icing is a complete interaction between environmental and aircraft characteristics. Although it's widely recognized that the three most important meteorological factors controlling ice accretion are LWC, temperature, and droplet size, the relative importance of these, and the interaction with aircraft-dependent parameters, such as airfoil shape, flight configuration, and deicing /anti-ice equipment, is still poorly understood. Thus, the lack of a simple relation between the environment and icing intensity makes it difficult to make accurate predictions of icing intensity. To actually estimate the icing severity level for a given aircraft, what is needed are the actual values of all the meteorological elements (temperature, LWC, MVD), the aircraft and flight parameters, and a sophisticated analysis or modeling technique (e.g. LEWICE) to evaluate the physics of the ice accretion process (Hansman, 1989).

2.3 Previous Studies. Determining the accuracy of in-flight icing forecasts is not straightforward, primarily due to limitations in the availability and quality of data to act as 'air truth' (Brown et al., 1993). The best available data would be from research aircraft designed to fly through areas of forecasted and known icing. However, many of the studies performed must depend on a less reliable data source — pilot reports (PIREPs) from commercial, military, and private pilots.

Forbes et al. (1977) tried using multiple regression techniques to predict icing intensity based on temperature, wind speed, and the lapse rate of relative humidity (RH) but had little success. They concluded that different thresholds of the RH lapse rate were needed under stable and conditionally unstable conditions. Abromovich et al. (1977) used multiple regression and discriminant analysis techniques to predict icing type and intensity. Although they concluded that you cannot establish clear boundaries of classification, they did have some success when

using liquid water content and the temperature and height of the base of the cloud as predictors. Bernhardt (1989) compared objective methods that used condensation supply rate (CSR) to subjective forecasts made by forecasters. The CSR method fared well with an 85 percent correct and 38 percent false alarm rate. He suggested that further comparisons be done between the CSR method and other objective techniques (e.g. Appleman's -8D & AWS techniques).

Currently, the FAA is in the middle of a six-year program to: (1) Perform basic research in the area of aircraft icing; (2) Develop and test numerical analysis systems to identify areas of icing; and (3) Develop a meteorological-based icing severity index (Politovich and Sand, 1991). AFCCC continues to monitor the results from this cooperative effort which involves a number of organizations.

2.4 Icing Climatologies. During the 1950s, Air Weather Service (AWS) flew a number of reconnaissance flights over the north Pacific and Atlantic oceans. The aircraft flew at the same levels (700 and 500mb) and on the same routes so it was possible to statistically analyze the results to obtain the conditional probability of aircraft icing given cloud amounts greater than 6/10, as a function of temperature and altitude (Appleman, 1959). This data set has been used in numerous studies, including Katz (1967), who used the results to forecast the probability of icing in 5000-foot layers from the surface to 20,000 ft. This study was based on limited data and does not address icing severity. Heath (1972) calculated the frequency of occurrence of icing for the northern hemisphere, correlating temperature-dewpoint differences with the probability of icing based on the AWS flight data. The results showed a high frequency of icing over southeastern Canada at 850mb (Probability (P) > 0.1), over Scandinavia and northeastern Russia (P > 0.15), and over the northern and western Pacific in January (P > 0.2). Again, there was

not enough data to guarantee full resolution of icing occurrence, and icing severity was not considered. AFCCC (1984) compiled an extensive LWC database covering the years 1977-1980, with LWC values calculated using the original Smith-Feddes model. From this data set, the joint probability of occurrence of LWC values greater than 1 gm^{-3} with temperatures between

0 and -40°C were calculated monthly and annually for eight vertical layers, at a 200 nautical mile resolution. Although, the Smith-Feddes model probably exhibits a bias towards overestimating LWC, the distribution of higher LWCs ($> 1 \text{ gm}^{-3}$) can be considered to delineate areas of severe icing.

Chapter 3

METHODOLOGY

3.1 General. Forecasts of aircraft icing type and intensity from three operational algorithms and a statistical classification procedure were compared to a database of pilot reports. In addition to an overall comparison, a stratified comparison was done based on aircraft characteristics of engine type, weight, and performance (i.e., climb rate).

3.2 Data.

3.2.1 PIREPs. A database of pilot reports (PIREPs) of aircraft icing from across the contiguous United States taken during the period from March 1990 to March 1991 was obtained from NOAA's Forecast Systems Lab (FSL) in Boulder, CO (Schultz and Politovich, 1992). The PIREPs consisted of aircraft type and location, and icing type and intensity. The nearest zero-hour analysis and 12-hour forecast for temperature and relative humidity from National Weather Service's Nested Grid Model (NGM) was appended to each PIREP. The PIREP database contains over 90,000 PIREPs, but only PIREPs that occurred within one hour of 00 or 12 UTC and whose intensity and type were known were used in this study. Additionally, because the forecast algorithms forecast fewer intensity categories than were reported, the reported intensities were consolidated to reflect the most severe condition. For example, if the PIREP indicated light-moderate intensity, this PIREP was grouped in with the PIREPs of moderate intensity.

In an effort to limit aircraft specific influences, information on engine type, weight class and climb rate were assigned to the PIREPs based on the reported aircraft designation and FAA Regulation 7110.65H. The engine types assigned were piston, turboprop, and jet. The weight

classes were small, light, and heavy. Aircraft performance was characterized by the FAA's specification of climb rate. Aircraft were categorized into three climb rate groups: < 2000 feet per minute (fpm), < 3000 fpm (but \geq 2000 fpm), and \geq 3000 fpm.

3.2.2 Upper-Air. Because the forecast algorithms require data in addition to that provided within the PIREPs (e.g. lapse rate) to determine the type and intensity of icing, upper-air soundings from AFCCC's DATSAV database were used by the forecast algorithms. Data from the closest site (temporally and spatially) of the NWS network of upper-air stations was appended to each PIREP.

3.2.3 Limitations. There are several drawbacks to using PIREPs and routine upper air soundings to validate icing forecasts. These include: (1) Reports of "no icing" are extremely rare, so null cases cannot be evaluated; (2) PIREPs are uncalibrated, meaning icing severity depends on the pilot's perception of what his aircraft can safely handle; (3) Reported position, altitude and time may be in error in the PIREPs; (4) PIREPs are not necessarily independent observations, but may result from requests for PIREPs (Politovich and Olsen, 1991); and (5) the PIREPs and upper air data are separated spatially and temporally, and the forecast routines require meteorological data not normally reported within PIREPs. This produced time and space discontinuities when comparing the icing conditions reported by the PIREPs and that calculated by the algorithms. However, given the nature of this study (i.e. a comparison between algorithms), we felt this PIREP database would suffice as 'air-truth.'

3.3 Icing Forecast Algorithms. Three operational icing forecast algorithms and discriminant analysis procedures were used to predict icing type and intensity: (1) the Air Force Global Weather Central's (AFGWC) RAOB icing routine; (2) the icing routine used within the Naval Oceanographic Data Distribution System (NODDS) software; and (3) the icing routine used in the Navy's Tactical Environmental Support System (TESS) software.

3.3.1 RAOB Icing Algorithm. AFGWC's icing routine uses temperature, dew point depression, and stability as measured by the temperature lapse rate to forecast icing type and intensity (Table 1). For this study, an icing type and intensity was assigned at each sounding level based on its temperature, dew point, and temperature lapse rate between the level and the next higher level. The bottom and tops of icing layers were determined by interpolating temperature and dew point levels with height to meet the icing criteria. The most severe type and intensity within an icing layer were then assigned to the layer as a whole. (Note: under no conditions does this algorithm forecast severe icing.)

3.3.2 NODDS Icing Algorithm. The NODDS icing routine differs from Global's routine in that it uses the average temperature and dew point depression in the icing layer and the Showalters Stability Index (SSI) to determine the icing type and intensity for the layer as a whole. For this study, the average temperature and dew point within a layer was determined by summing the

temperatures and dew points through the layer and dividing by the number of levels within the layer. Therefore, no attempt was made to weight the components of the average for the varying distances between levels within a layer. The average dew point depression was then the difference between the average temperature and average dew point. Table 2 is NODDS's decision matrix for forecasting icing.

3.3.3 TESS Icing Algorithm. The TESS algorithm was developed from the nomogram for icing contained in AWS/TR—80/001, *Forecasters Guide on Aircraft Icing*. This algorithm assigns an icing type and intensity along with the probability of icing to each level meeting specified criteria. These elements are assigned based on: (1) height above cloud base; (2) atmospheric stability; (3) temperature and dew point depression; and (4) the presence of a frontal inversion. Table 3 specifies the decision matrix for assigning icing probability and type. Icing intensity is taken from two look-up tables. These tables classify the icing intensity as a function of the temperature at the level, the distance above the cloud base, the icing type, and whether the layer contains a frontal inversion. For this study, if the probability of icing was greater than zero then icing was forecast. As with the RAOB algorithm, the bottoms and tops of icing layers were determined by interpolating temperature and dew point levels with height to meet the icing criteria; the most severe type and intensity within an icing layer was used to describe the layer as a whole.

Table 1. Decision matrix for forecasting icing type and intensity used by RAOB.

Temperature, ° C	$0 \geq T > -8$				$-8 \geq T > -16$				$-16 \geq T \geq -22$
Dew Point Depression, ° C ($T-T_d$)	≤ 1		$1 < T-T_d \leq 2$		≤ 1		$1 < T-T_d \leq 3$		≤ 4
Lapse Rate ° C/ 1000 ft	≤ 2	> 2	≤ 2	> 2	≤ 2	> 2	≤ 2	> 2	N/A
Forecast Icing	Lgt Rime	Mdt Clear	Trace Rime	Lgt Clear	Mdt Rime	Mdt Mixed	Lgt Rime	Lgt Mixed	Lgt Rime

Table 2. Decision matrix for forecasting icing type and intensity used by NODDS.

	Average Temperature, $T_{avg} > -12$						
$T-T_d$	$3 \leq T-T_d \leq 4$			$1 \leq T-T_d < 3$			< 1
SSI	$3 \leq SSI \leq 4$	$2 \leq SSI < 3$	< 2	$3 \leq SSI \leq 4$	$1 \leq SSI < 3$	< 1	< 0
Forecast Icing	Trace Clear	Lgt Clear	Mdt Clear	Lgt Clear	Mdt Clear	Svr Clear	Svr Clear
	Average Temperature, $T_{avg} > -12$						
$T-T_d$	$3 \leq T-T_d \leq 4$		$2 \leq T-T_d < 3$		$1 \leq T-T_d < 2$		< 1
SSI	> 4						
Forecast Icing	Trace Rime		Lgt Rime		Mdt Rime		Svr Rime
	Cloud Base Temperature < -10 or $-22 < T_{avg} \leq -12$						
$T-T_d$	$3 \leq T-T_d \leq 4$		$2 \leq T-T_d < 3$		$1 \leq T-T_d < 2$		< 1
SSI	$0 < SSI \leq 2$				$-2 \leq SSI \leq 0$		< -2
Forecast Icing	Trace Mixed		Lgt Mixed		Mdt Mixed		Svr Mixed

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Table 3. Matrix used in TESS to determine icing probability (Prob) and type base on temperature (Temp, ° C), dew point depression ($T - T_d$), lapse rate (° C/100m).

Temperature	$0 \geq T \geq -7$			
$T - T_d$	$2 \leq T - T_d \leq 4$		< 2	
Probability	20		100	
Lapse Rate	$< .55$	$\geq .55$	$< .55$	$\geq .55$
Icing Type	Rime	Mixed	Rime	Clear
Temperature	$-7 > T \geq -15$			
$T - T_d$	$3 \leq T - T_d \leq 6$		< 3	
Probability	20		100	
Lapse Rate	$< .55$	$\geq .55$	$< .55$	$\geq .55$
Icing Type	Rime	Mixed	Rime	Clear
Temperature	$-15 > T \geq -22$			$-22 \geq T \geq -30$
$T - T_d$	$4 \leq T - T_d \leq 6$	< 4		< 6
Probability	10	100		10
Lapse Rate	N/A	$< .55$	$\geq .55$	N/A
Icing Type	Rime	Rime	Clear	Rime

3.3.4 Discriminant Analysis Classification of Icing. Discriminant analysis is a statistical procedure that classifies individual observations into groups. It is a supervised classification in that we identify the groups into which we would like our observations classified. In this case, given our predictor variables of temperature, moisture, and stability, the discriminant attempts to group observations into three classes of icing type (clear, mixed, rime) and three classes of icing intensity (trace, light, moderate). Several different combinations of the predictor variables were attempted and an overall classification accuracy was determined for each. The classification accuracies are based on results using the CROSSVALIDATE option with SAS's (SAS Institute, Cary, NC) DISCRIM procedure. This option is a way of obtaining nearly unbiased estimators of prediction error when it is not practical to test the discriminant on an independent data set.

3.4 Comparison Methodology. Data sets containing the forecasted icing layers were merged with the PIREPs by upper-air station, date, and time of day. A comparison was made between the forecast and observed icing using the following queries:

1. Does the PIREP flight level fall within the forecast icing layer? If so:
 - a. Do the icing types agree?
 - b. Do the icing intensities agree?
 - c. Do both the type and intensity agree?

2. Icing is forecast but observed outside the forecast layer.

- a. Do the icing types agree?
 - b. Do the icing intensities agree?
 - c. Do both the type and intensity agree?
 - d. What is mean distance above forecast icing layer?
 - e. What is mean distance below forecast icing layer?

3. How many times was icing forecasted but not observed?

4. How many times was icing observed but not forecasted?

A comparison was also made by inputting the NGM temperature and moisture analysis data, rather than the rawinsonde data, into the algorithms. This revealed any bias we may have introduced by assigning the most severe icing level forecast to the layer as a whole when using the RAOB and TESS forecast algorithms. Of course, using flight level data with the NODDS algorithm, which is designed for layer averaged data, adds an artificiality there. Also of interest, are differences that may be associated with aircraft type. Subsequently, the database was stratified according to aircraft characteristics used by the FAA to classify aircraft, i.e. engine type (piston, turboprop, jet), weight (small, light, heavy), and climb rate prior to performing this comparison.

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3.5 Descriptive Statistics. In addition to the icing forecast algorithm comparison, some simple descriptive statistical summaries were determined for the forecasted and observed icing conditions. These included:

1. Frequency of occurrence of icing type and intensity stratified by icing type, intensity, and aircraft characteristics (engine type, aircraft weight, and climb rate).
2. Mean base, top, and thickness of forecasted icing layers.

3. Frequency distributions and/or means and standard deviations of temperature, dew point depression, relative humidity, flight level liquid water content (LWC), and mean LWC as function of icing type and intensity. LWC was determined using an algorithm recently adopted by AFCCC (Cornell and Donahue, 1994). Mean LWC was determined by averaging the LWC values over the extent of the cloud. Clouds were identified using a criteria of 75 percent relative humidity.

Chapter 4

RESULTS

4.1 Summary of PIREPs. A total of 9,693 PIREPS were used in this study. A total of 5093 upper-air soundings from 75 different upper-air sites were analyzed for icing using the three operational forecast algorithms. The date and time groups occurred on a total of 680 different dates and times during the March 1990-March 1991 time period. The average distance between a PIREP and the closest upper-air site was 74 miles. There were 88 occurrences where an upper-air sounding was not available.

Tables 4 and 5 provide a summary of the final PIREP data set. The majority of the icing reports were for trace or light rime icing. Table 4 gives the mean and standard deviation for the flight level, temperature, and moisture characteristics for the different icing types and intensities. The frequencies of icing type (78 percent rime, 6 percent clear, 16 percent mixed) are very close to those reported in a summary of Air Force reconnaissance flights by Perkins et al. (1957) (72 percent rime, 10 percent clear, 17 percent mixed). Rime icing is associated with the coldest temperatures and widest dew point depression. Mixed icing occurs at temperatures between rime and clear but has the lowest temperature-dew

point spread. The mean dew point depression for all types and intensities is 4.5°C and the mean relative humidity is 70 percent. Table 5 provides a further breakout of the frequency of occurrence of type and intensity by aircraft characteristics. Furthermore, the faster and heavier aircraft report rime and moderate icing at a higher frequency than the smaller, slower aircraft.

Means and standard deviations of flight level, temperature, dew point depression, Showalters stability index (SSI), and liquid water content by reported icing type and intensity for all aircraft combined and stratified by aircraft characteristics are provided in Appendix A. Of particular interest here is the tendency for clear ice to occur at more stable conditions (as measured by the lapse rate and SSI) than rime ice. The tendency for jet aircraft to experience clear icing at colder temperatures (mean = -10.2 °C) than slower aircraft (mean = -3.8 °C) is clearly evident. For the most part, higher performance aircraft experience the same intensity icing as lower performance aircraft at a higher flight level, colder temperatures, larger dew point depressions, and less stable conditions.

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Table 4. Frequency (N) and percent frequency (%) of occurrence of icing type and intensity and mean and standard deviation of flight level, temperature, dew point depression, and relative humidity for PIREPs used in comparison study. Temperature and relative humidity are from the NGM analysis and the dew point depression was determined from these values.

		N	%	Mean \pm Standard Deviation			
				Flight Level (m MSL)	Temperature ° C	Dew Point Depression	Relative Humidity (%)
Icing Type	Clear	589	6	2797 \pm 1676	-6.6 \pm 6.2	4.5 \pm 4.4	71 \pm 18
	Mixed	1535	16	3210 \pm 1741	-8.8 \pm 6.3	4.2 \pm 3.4	71 \pm 16
	Rime	7569	78	3513 \pm 1803	-9.6 \pm 7.0	4.6 \pm 3.6	69 \pm 18
Icing Intensity	Trace	5209	54	3337 \pm 1739	-8.8 \pm 7.1	4.6 \pm 3.5	69 \pm 18
	Light	4152	43	3516 \pm 1854	-9.8 \pm 6.6	4.5 \pm 3.6	69 \pm 18
	Moderate	332	3	3568 \pm 1878	-10.5 \pm 6.7	4.4 \pm 3.8	70 \pm 19

Table 5. Frequency and percent frequency of type and intensity of icing by aircraft engine type, weight class, and climb rate.

		Engine Type			Weight Class			Climb Rate (CR)		
		Piston	Turboprop	Jet	Small	Light	Heavy	<2000	2000<CR<3000	>3000
Icing Type	Clear	222 8%	186 5%	143 5%	325 7%	226 5%	0	195 8%	221 5%	135 6%
	Mixed	400 15%	698 20%	300 10%	777 17%	618 14%	3 9%	397 16%	724 16%	277 13%
	Rime	2045 77%	2651 75%	2428 85%	3528 76%	3567 81%	29 91%	1951 76%	3475 79%	1698 81%
Icing Intensity	Trace	1592 60%	2040 58%	1214 42%	2829 61%	2001 45%	16 50%	1489 59%	2435 55%	922 44%
	Light	985 37%	1448 41%	1493 52%	1677 36%	2233 51%	16 50%	979 38%	1868 42%	1079 51%
	Moderate	90 3%	47 1%	164 6%	124 3%	177 4%	0	75 3%	117 3%	109 5%

4.2 Summary of Icing Forecasts.

4.2.1 Analysis of Upper-air. Tables 6-8 outline the forecasts made by the three operational forecast algorithms using the available upper-air data. For this analysis, the most severe condition found within a layer is assigned to that layer as a whole for the RAOB and TESS algorithms which assign an icing type and intensity to each sounding level. On the other hand, NODDS uses the average values within a layer to assign an icing type and intensity to the layer. Overall, TESS forecasted the most icing layers (TESS — 7,067, RAOB — 5,328, NODDS — 4,697). The average icing base was at 3,216 meters (m) for

TESS, 2,834 m for RAOB, and 2,610 m for NODDS with the average icing layer being 1,541 m thick according to TESS, 1,240 m according to RAOB, and 1,420 m according to NODDS. The distributions of type and intensity do not compare very well with the PIREP's distribution (Table 4). TESS and RAOB forecasted clear icing at a high rate — 56 percent and 36 percent respectively. This could be a result of assigning the most severe level value to the layer as a whole. TESS forecasted severe 37 percent of the time and NODDS forecasted severe at a 1 percent rate. None of the PIREPS reported severe and RAOB does not forecast severe.

Table 6. Frequency (N) and percent frequency (%) of occurrence of icing type and intensity and mean and standard deviation of base height, top height and thickness of icing layers as forecasted by NODDS using rawinsonde data. Heights are meters (m) above mean sea level.

		N	%	Mean \pm Standard Deviation		
				Base Height (m)	Top Height (m)	Thickness (m)
Icing Type	Rime	2559	55	1747 \pm 1252	3347 \pm 1878	1601 \pm 1394
	Clear	416	9	2778 \pm 1151	4239 \pm 1588	1462 \pm 1185
	Mixed	1722	36	3854 \pm 1718	4995 \pm 1730	1141 \pm 1026
Icing Intensity	Trace	1405	30	3432 \pm 1871	4064 \pm 1949	632 \pm 745
	Light	2287	49	2509 \pm 1603	4054 \pm 1954	1544 \pm 1125
	Moderate	964	20	1732 \pm 1271	4003 \pm 1943	2271 \pm 1502
	Severe	41	1	769 \pm 804	2235 \pm 2248	1466 \pm 1816

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Table 7. Frequency (N) and percent frequency (%) of occurrence of icing type and intensity and mean and standard deviation of base height, top height and thickness of icing layers as forecasted by RAOB using rawinsonde data. Heights are meters (m) above mean sea level.

		N	%	Mean \pm Standard Deviation		
				Base Height (m)	Top Height (m)	Thickness (m)
Icing Type	Rime	2054	38	3459 \pm 2006	4568 \pm 2098	1109 \pm 1194
	Clear	1904	36	2011 \pm 1256	3248 \pm 1566	1237 \pm 1165
	Mixed	1370	26	3040 \pm 1586	4480 \pm 1727	1440 \pm 1224
Icing Intensity	Trace	273	14	2203 \pm 1207	2707 \pm 1263	503 \pm 431
	Light	4992	81	2956 \pm 1836	4177 \pm 1952	1221 \pm 1202
	Moderate	763	5	2372 \pm 1473	3980 \pm 1799	1608 \pm 1228

Table 8. Frequency (N) and percent frequency (%) of occurrence of icing type and intensity and mean and standard deviation of base height, top height and thickness of icing layers as forecasted by TESS using rawinsonde data. Heights are above mean sea level.

		N	%	Mean \pm Standard Deviation		
				Base Height (m)	Top Height (m)	Thickness (m)
Icing Type	Rime	2355	33	4022 \pm 2492	4895 \pm 2501	872 \pm 9014
	Clear	3947	56	2692 \pm 1618	4844 \pm 2170	2152 \pm 1637
	Mixed	765	11	3431 \pm 1642	3880 \pm 1718	449 \pm 480
Icing Intensity	Trace	1695	24	4430 \pm 2110	4803 \pm 2128	372 \pm 389
	Light	1698	24	3779 \pm 2278	4818 \pm 2548	1039 \pm 720
	Moderate	1041	15	2353 \pm 1640	3814 \pm 2267	1461 \pm 1116
	Severe	2633	37	2411 \pm 1378	5060 \pm 2045	2649 \pm 1709

4.2.2 Algorithms Applied at Flight Level. In this analysis, the algorithms were used with the temperature and moisture profiles from the NGM analysis. Lapse rate and SSI data were still derived from the upper-air DATSAV data. An analysis of the differences between temperature and relative humidities from the upper-air database and the NGM analysis data showed that 70 percent of these differences were within 2° C for temperature and 20 percent for relative humidity. Although this analysis more accurately reflects what TESS and RAOB would have forecast at flight level, it wrongly utilizes unaveraged data with NODDS. Table 9 provides the results from this analysis in terms of the forecast frequencies. Intensities using the TESS look-up tables could not be practically determined for this analysis.

A comparison of this analysis to the upper-air forecast (Tables 6-8) shows: (1) A sharp increase in forecasts of clear icing and a decrease in the forecasts of mixed icing by NODDS; (2) A drastic decrease in the forecasts of clear and mixed icing

by RAOB; (3) Fewer clear forecasts, but more mixed icing forecasts by TESS; (4) More moderate and severe forecasts by NODDS; and (5) More trace and less light icing by RAOB.

4.3 Icing Comparison.

4.3.1 Forecasts from Upper-air. Table 10 summarizes the results of the comparison between forecasted icing conditions based on analysis of upper-air data. In all cases, approximately 33 percent of the forecasted icing layers had a PIREP within the forecasted layer. In forecasting type, NODDS performed best with 46 percent correctly forecast. Intensity forecasts were most accurate with RAOB; 30 percent of the forecasts agreed with the observed condition. NODDS had the highest frequency of occurrence of not forecasting any icing on a day that icing occurred. Only on a few occurrences (1-3 percent) did the algorithms forecast icing and there was no icing report in a PIREP at that time. The distances between the PIREP flight level and the top/bottom of the forecasted icing layers are surprisingly high.

Table 9. Frequency (N) and percent frequency (%) of occurrence of icing type and intensity as forecasted by indicated algorithm. Forecast based on NGM analysis of flight level temperature and moisture and closest upper-air sounding for determination of lapse rate and Showalters stability index.

		NODDS		RAOB		TESS	
		N	%	N	%	N	%
Icing Type	Rime	486	30	5	<1	601	25
	Clear	42	3	20	2	583	24
	Mixed	1087	67	1150	98	1253	51
Icing Intensity	Trace	558	27	279	24	N/A	
	Light	503	24	785	67		
	Moderate	678	33	111	9		
	Severe	320	16	N/A	N/A		

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Table 10. Results of comparison between three icing forecast algorithms (NODDS, RAOB, and TESS) and pilot reports (PIREPs) of icing over the contiguous United States during the March 1990-March 1991 time period.

		NODDS	RAOB	TESS
Level of PIREP in forecasted icing layer (Percentage is of total PIREPS whose flight level fell within forecasted icing layer)	Types agree	880 (46%)	479 (26%)	373 (16%)
	Intensities agree	493 (26%)	587 (31%)	165 (7%)
	Both type and intensity agree	224 (12%)	162 (9%)	69 (3%)
	Neither type or intensity agree	745 (39%)	963 (52%)	1906 (80%)
Level of PIREP outside of forecasted icing layer (Percentage is of total PIREPS whose flight level fell outside icing layer)	Types agree	1832 (41%)	1750 (32%)	1899 (30%)
	Intensities agree	1515 (34%)	1759 (33%)	1486 (24%)
	Both type and intensity agree	650 (14%)	714 (13%)	724 (11%)
	Neither type or intensity agree	1793 (40%)	2604 (48%)	3662 (58%)
	Mean distance above forecasted layer (m)	1928 (58%)	1903 (55%)	1873 (46%)
	Mean distance below forecasted layer	1615 (42%)	1623 (45%)	1922 (54%)
No icing forecasted at time of PIREP (Percentage is of total number of PIREPs)		2149 (22%)	1320 (14%)	600 (6%)
No PIREP reported for time of icing forecast (Percentage is of total number of forecasted layers)		44 (1%)	10 (<1%)	212 (3%)

4.3.2 Flight Level Forecasts. Tables 11 and 12 provide a summary of comparisons between PIREP data and icing forecasts made from inputting flight level temperature and moisture data (derived from the NGM analysis) and stability information from upper-air soundings into the forecast algorithms. Table 11 gives an overall comparison broken out by observed type and intensity. The algorithms have a hard time forecasting observed occurrences of clear icing. The TESS algorithm, which forecasts clear at the highest rate still only matched 20 percent of the clear PIREPs. All the algorithms assume that clear icing occurs under unstable conditions. However, the data used here indicates that clear icing occurs under conditions of higher stability (in terms of lapse rate and SSI) than rime icing. What the algorithms fail to account for is that although clear icing does occur under unstable conditions (e.g. in thunderstorms) it also can be expected under very stable conditions in which large drops are likely to be present (Politovich, 1989). Therefore, the lack of agreement between forecasted and observed clear icing can be attributed to: (1) a lack of PIREPS taken in conditions of instability due to avoidance of these conditions by aircraft, and (2) the forecast algorithms forecast clear icing only under unstable conditions. For intensity, RAOB forecasted moderate on only 2 percent of the occasions when it was observed, whereas NODDS had a 46 percent rate of agreement for moderate icing. RAOB's dew point depression criteria may be too stringent (Table 1); NODDS forecasts moderate icing with dew point depressions as high as 4 degrees when conditions are unstable (Table 2).

Overall, RAOB had the highest percentage of agreement for both type and intensity but this may be more a reflection of the PIREPs having so few observations of clear/mixed type and moderate intensity than of a better forecast algorithm used in the RAOB. One additional advantage of the RAOB algorithm over the NODDS is that it will differentiate icing of different type and intensity within the same cloud (icing layer), while NODDS uses the average conditions in the cloud to specify a single type and intensity for the cloud as a whole. Certainly, an aircraft can expect more variable icing conditions than those implied by the NODDS algorithm.

Table 12 breaks out the percentages of agreement between forecasted and observed conditions by aircraft characteristics of engine type, weight class, and climb rate. Its interesting to note that this stratification impacts RAOB's percentages much more than the other forecast algorithms. The frequency of agreement between RAOB's forecast and observed conditions actually increases quite a bit when considering only PIREPs from faster and heavier (light vs small) aircraft. Some of this may be due to sampling differences. Due to inconsistencies in taking and archiving upper-air data (e.g. how high the balloon goes, data lost in transmission), we were able to calculate SSI on over 2000 more observations than those for which we could get flight level lapse rate information. Subsequently, because of differences in data availability, each comparison is performed with a somewhat different subset of PIREP and upper-air data.

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Table 11. Frequency and percent frequency of agreement between observed and forecasted icing type and intensity as function of observed type (TYP) and intensity (INT). Types and intensity forecast based on NGM analysis of temperature and dew point at flight level and calculation of lapse rate and Showalters stability index from rawinsonde data.

Icing Forecast Algorithm	All Observations		Observation Type						Observation Intensity					
			Rime		Clear		Mixed		Trace		Light		Moderate	
	TYP	INT	TYP	INT	TYP	INT	TYP	INT	TYP	INT	TYP	INT	TYP	INT
ROAB	782 67%	497 42%	757 82%	389 42%	5 7%	34 46%	20 12%	74 43%	407 70%	148 25%	338 63%	347 65%	37 69%	2 4%
NODDS	1161 56%	568 28%	1131 73%	416 27%	19 13%	49 34%	11 3%	103 28%	653 58%	297 27%	469 55%	233 27%	39 48%	38 46%
TESS	1109 46%	N/A	959 51%	N/A	28 20%	N/A	122 30%	N/A	570 46%	N/A	497 45%	N/A	42 46%	N/A

Table 12. Frequency and percent frequency of agreement between observed and forecasted icing type and intensity as function of aircraft characteristics as recorded in FAA regulation 7110.65. Type (TYP) and intensity (INT) forecasts based on NGM analysis of temperature and dew point at flight level and calculation of lapse rate and Showalter's stability index from rawinsonde data.

Icing Forecast Algorithm	Engine Type						Weight Class				Climb rate in feet per minute					
	Piston		Turboprop		Jet		Small		Light		< 2000		< 3000		≥ 3000	
	TYP	INT	TYP	INT	TYP	INT	TYP	INT	TYP	INT	TYP	INT	TYP	INT	TYP	INT
ROAB	219 63%	125 36%	301 67%	195 43%	196 73%	143 53%	350 63%	213 38%	366 72%	250 49%	183 59%	118 38%	380 71%	248 46%	153 70%	97 44%
NODDS	349 58%	157 26%	475 57%	237 29%	249 53%	138 30%	585 56%	267 26%	488 57%	265 31%	345 57%	165 27%	545 59%	265 29%	183 49%	102 27%
TESS	303 43%	N/A	413 46%	N/A	293 47%	N/A	506 45%	N/A	503 47%	N/A	275 43%	N/A	520 47%	N/A	214 45%	N/A

4.4 Discriminant Analysis. AFCCC/SYT used discriminant analysis to classify observations according to their type and intensity. These classification accuracies for aircraft stratified data are given in Tables 13 and 14. Classification accuracies prior to stratification can be estimated by averaging. These results clearly show the difficulty in trying to assign an icing type and intensity from the indicated variables, even when taking into consideration aircraft type. The inclusion of lapse rate does improve the classification of icing type. However, as shown in Figure 1, the observed relationship between lapse rate and type is opposite that reflected in the operational algorithms (the more stable it is, the more likely the icing type will be clear). It is also clear from Table 13 that the lapse rate at flight level is a better discriminator of icing type than the Showalters stability index. Figure 2 shows the distribution of temperature and lapse rate as a function of icing intensity. The similarity of the distribution makes it difficult (if not impossible) to differentiate between icing

intensities based solely on temperature and lapse rate. The indications from Table 14 are that the variables used by TESS to determine icing intensity perform more consistently across aircraft type than the other predictors tried. However, it should be emphasized that the liquid water content and median volume diameter are simple empirical parameterizations and most likely do not reflect the actual LWC and MVD at the time the PIREP was taken. The consensus throughout the scientific community is that without realistic parameterizations of LWC and drop size distribution, we cannot expect much improvement over the present forecast methods. It has been stated that 85 percent of the observed aircraft icing occurs in the vicinity of frontal zones (AWS, 1980). Table 15 summarizes the frequency of occurrence the upper-air data indicated the presence of a frontal inversions as a function of icing type and intensity. Higher frequencies of frontal inversions are associated with clear versus rime and moderate versus trace icing.

Table 13. Overall classification accuracy for classifying icing type based on stated prediction variables. Data stratified by aircraft characteristics.

Prediction Variables	Engine Type			Climb Rate (ft per minute)		
	Piston	Turboprop	Jet	< 2000	< 3000	> 3000
Temperature	30	27	23	30	27	28
Temperature & Lapse Rate	36	62	34	33	60	32
Temperature & Lapse rate of θ_e	27	36	34	29	34	36
Temperature & SSI	31	26	20	28	27	32
Temperature & Lapse Rate & Dew Point Depression	40	54	34	38	61	32
Temperature & Lapse rate of θ_e & Dew Point Depression	28	42	34	31	36	36

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Table 14. Overall classification accuracy for classifying icing intensity based on stated prediction variables. Median volume diameter assigned based on cloud type. Data stratified by aircraft characteristics.

Prediction Variables	Engine Type			Climb Rate (ft per minute)		
	Piston	Turboprop	Jet	< 2000	< 3000	> 3000
Liquid Water Content (LWC)	12	19	34	18	42	16
LWC & MVD	17	10	38	16	52	23
Mean LWC	16	42	22	18	38	15
Mean LWC & MVD	26	10	40	17	43	40
Dew Point Depression & Temperature Lapse Rate	16	46	34	18	39	19
Dew Point Depression & Lapse Rate θ_e	18	46	50	17	40	26
Dew Point Depression & SSI	22	28	36	21	20	35
Temperature & Height & Icing Type & Presence of Frontal Inversion (Y/N)	46	45	38	44	48	40
Temperature & Height above cloud base & Icing Type & Presence of Frontal Inversion	40	47	30	37	45	51

Table 15. Frequency and percent frequency of occurrence of frontal inversions as a function of icing type and intensity.

		Yes	No
Icing Type	Clear	349 (63%)	202 (37%)
	Mixed	844 (61%)	544 (39%)
	Rime	4190 (60%)	2741 (40%)
Icing Intensity	Trace	2844 (59%)	1935 (41%)
	Light	2339 (62%)	1458 (38%)
	Moderate	200 (66%)	104 (34%)

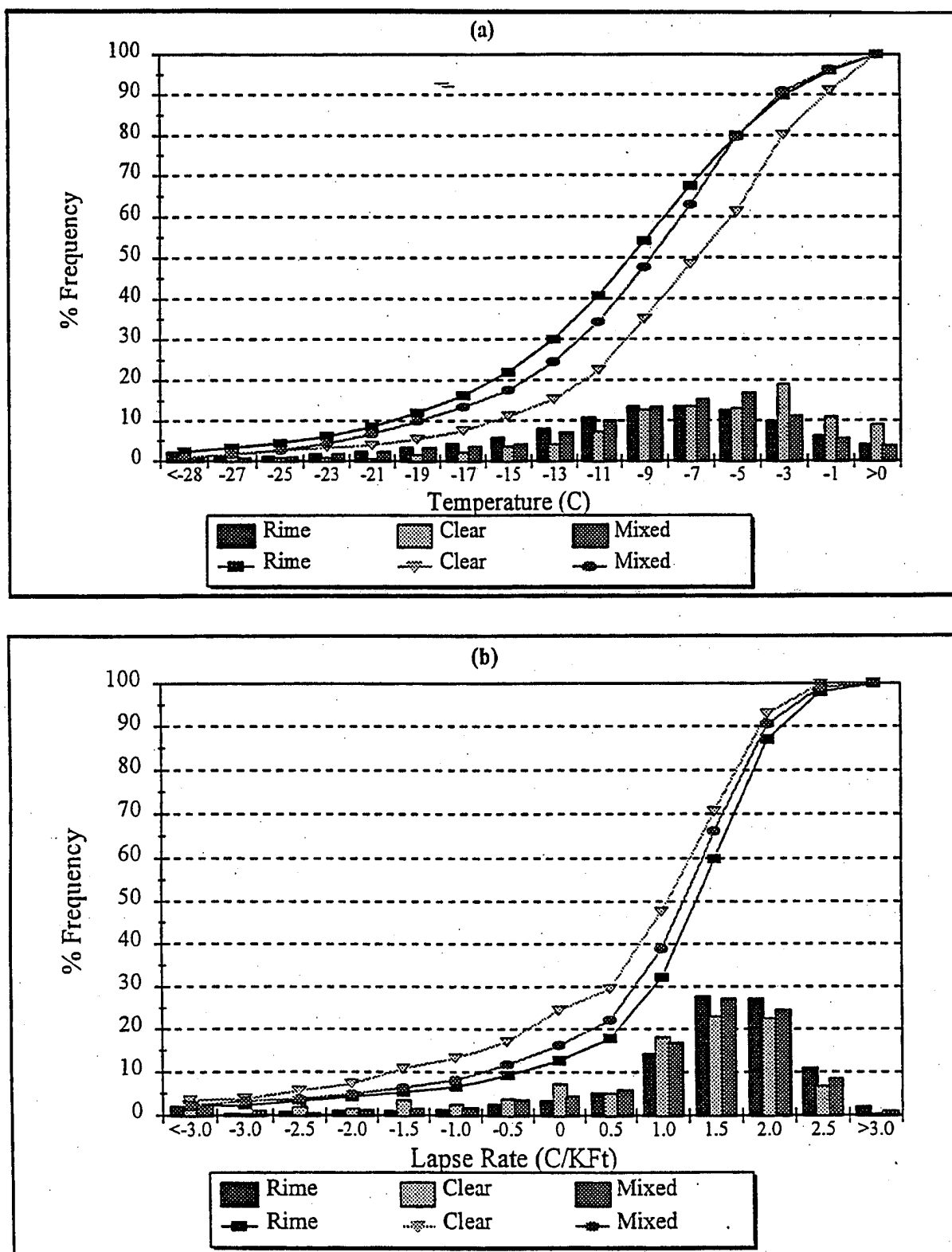


Figure 1. Percent and cumulative percent frequency of occurrence of icing type as a function of (a) temperature and (b) lapse rate. The temperature data is binned in 2 degree intervals, the lapse rate data in 0.5 degree intervals.

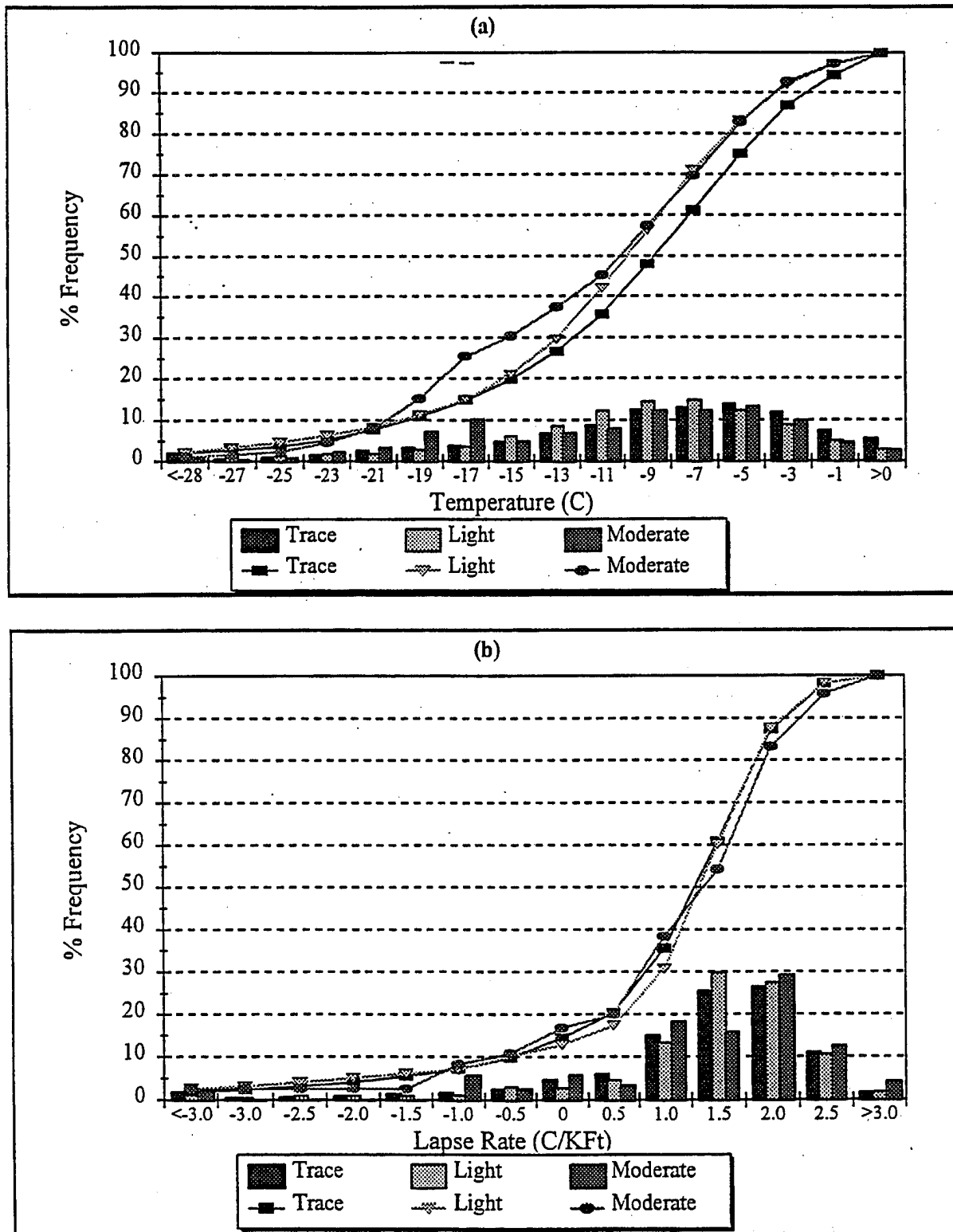


Figure 2. Percent and cumulative percent frequency of occurrence of icing intensity as a function of (a) temperature and (b) lapse rate. The temperature data is binned in 2 degree intervals, the lapse rate data in 0.5 degree intervals.

Chapter 5

SUMMARY AND CONCLUSIONS

5. Summary and Conclusions. AFCCC/SYT performed this study to compare forecasts of aircraft icing type and intensity from three operational forecast algorithms and discriminant analysis procedures to PIREPs of icing. Along with an overall comparison, the validation data was stratified by aircraft characteristics of engine type, weight class, and performance (climb rate).

PIREPs verified approximately 33 percent of the icing layers forecasted by the operational algorithms when using rawinsonde data as inputs. Of these, only 12 percent of the forecasts made with the Navy's NODDS icing routine agreed with the PIREP in terms of both icing type and intensity. However, this was the highest percentage agreement of the three operational algorithms (AFGWC's RAOB icing routine, NODDS, and the Navy's TESS icing routine) tested. However, when flight level data was inputted directly into the operational algorithms, RAOB performed best with 67 percent of the type forecasts and 42 percent of the intensity forecasts agreeing with the reported conditions. These percentages increase to 73 percent and 53 percent respectively when considering PIREPs only from jet aircraft.

All the algorithms failed to forecast clear or mixed icing under the stable atmospheric conditions that a large percentage of the clear and mixed icing PIREPs were associated with. The three operational algorithms assume that clear or mixed icing is associated with unstable conditions and rime icing occurs under stable conditions.

However, clear/mixed type icing can also be expected to occur under very stable conditions with large ($> 50 \mu\text{m}$) drop sizes present and pilots can be expected to avoid unstable atmospheric conditions. Results from a purely statistical classification technique (discriminant analysis) further reflected the difficulty in differentiating the type and intensity of icing based on measured meteorological variables with classification accuracies no better than that which can be expected by chance.

We believe that the percentages of agreement between the RAOB icing routine and PIREPs for the occurrence of aircraft icing are the best that can be expected from a forecast algorithm based on temperature, dew point depression, and temperature lapse rate. Without more rigorous parameterizations of liquid water content and drop size distributions, we cannot expect to improve our ability to forecast aircraft icing even when taking into consideration aircraft characteristics. The FAA recognizes this and as a result, in 1990 they began a 6-year, multimillion dollar effort to develop improved icing forecast methods and devise a new icing severity index (Politovich and Sand, 1991). AFCCC will continue to monitor the results of this effort and may revisit this issue as new algorithms are developed and more reliable 'air-truth' data becomes available. For the present time, climatologies of aircraft icing type and intensity produced by AFCCC will be based on AFGWC's RAOB icing forecast algorithm.

APPENDIX A

Table A1. Mean and standard deviation of flight level, temperature, dew point depression ($T-T_d$), lapse rate, Showalters stability index (SSI), and liquid water content (LWC). Temperature and dew point depression are from NGM analysis closest to time of PIREP, whereas, lapse rate, SSI, and LWC are derived from the closest upper-air sounding.

		Flight Level (m MSL)	Temperature (° C)	($T-T_d$) (° C)	Lapse Rate (°C/1000 ft)	SSI	LWC (gm ⁻³)
Icing Type	Clear	2789 ± 1671	-6.6 ± 6.2	4.5 ± 4.4	.93 ± 2.0	10.7 ± 7.3	.28 ± .29
	Mixed	3197 ± 1737	-8.8 ± 6.3	4.2 ± 3.4	1.3 ± 1.5	9.2 ± 6.7	.35 ± .35
	Rime	3496 ± 1797	-9.6 ± 7.0	4.6 ± 3.6	1.5 ± 1.5	9.3 ± 6.7	.36 ± .37
Icing Intensity	Trace	3320 ± 1734	-8.8 ± 7.1	4.6 ± 3.5	1.4 ± 1.5	9.4 ± 6.9	.34 ± .35
	Light	3505 ± 1846	-9.8 ± 6.6	4.5 ± 3.6	1.4 ± 1.6	9.3 ± 6.6	.37 ± .38
	Moderate	3556 ± 1880	-10.5 ± 6.7	4.4 ± 3.8	1.4 ± 1.5	9.9 ± 6.2	.31 ± .34

Table A2. Mean and standard deviation of flight level, temperature, dew point depression ($T-T_d$), lapse rate, Showalters stability index (SSI), and liquid water content (LWC) for indicated icing type and aircraft engine type. Temperature and dew point depression are from NGM analysis closest to time of PIREP, whereas, lapse rate, SSI, and LWC are derived from the closest upper-air sounding.

Icing Type	Engine Type	Flight Level (m MSL)	Temperature (° C)	($T-T_d$) (° C)	Lapse Rate (°C/1000 ft)	SSI	LWC (gm ⁻³)
Clear	Piston	2191 ± 1238	-3.8 ± 5.2	5.4 ± 6.3	.96 ± 1.7	11.3 ± 7.2	.29 ± .26
	Turboprop	2769 ± 1596	-6.6 ± 5.9	3.9 ± 3.5	.82 ± 2.2	11.3 ± 7.3	.28 ± .25
	Jet	3732 ± 1908	-10.2 ± 5.8	4.2 ± 2.3	1.0 ± 2.6	9.1 ± 7.2	.27 ± .46
Mixed	Piston	2461 ± 1198	-6.6 ± 4.9	3.9 ± 3.2	1.3 ± 1.7	9.8 ± 6.1	.37 ± .40
	Turboprop	3153 ± 1590	-8.5 ± 5.8	4.4 ± 3.7	1.2 ± 1.6	9.3 ± 7.1	.34 ± .31
	Jet	4319 ± 2137	-12.1 ± 7.6	4.0 ± 2.9	1.5 ± 1.1	7.9 ± 5.9	.35 ± .38
Rime	Piston	2678 ± 1237	-7.0 ± 5.7	4.6 ± 3.4	1.3 ± 1.9	9.6 ± 6.8	.39 ± .39
	Turboprop	3534 ± 1714	-9.3 ± 6.5	4.4 ± 3.3	1.5 ± 1.3	9.6 ± 6.8	.32 ± .37
	Jet	4200 ± 1990	-12.1 ± 7.8	4.9 ± 4.1	1.6 ± 1.2	8.7 ± 6.5	.39 ± .35

Table A3. Mean and standard deviation of flight level, temperature, dew point depression ($T-T_d$), lapse rate, Showalters stability index (SSI), and liquid water content (LWC) for indicated icing type and aircraft weight class. Temperature and dew point depression are from NGM analysis closest to time of PIREP, whereas, lapse rate, SSI, and LWC are derived from the closest upper-air sounding. Data for heavy aircraft is from small sample and none of these aircraft reported clear or mixed icing.

Icing Type	Weight Class	Flight Level (m MSL)	Temperature ($^{\circ}\text{C}$)	($T-T_d$) ($^{\circ}\text{C}$)	Lapse Rate ($^{\circ}\text{C}/1000\text{ ft}$)	SSI	LWC (gm^3)
Clear	Small	2355 ± 1374	-4.4 ± 5.3	4.5 ± 5.4	$.78 \pm 2.0$	11.3 ± 7.2	$.26 \pm .24$
	Light	3406 ± 1856	-9.3 ± 6.1	4.6 ± 3.1	1.2 ± 2.2	9.9 ± 7.4	$.33 \pm .36$
Mixed	Small	2846 ± 1519	-7.6 ± 5.8	4.2 ± 3.7	1.2 ± 1.8	10.2 ± 7.0	$.35 \pm .36$
	Light	3643 ± 1899	-10.1 ± 6.7	4.2 ± 3.0	1.4 ± 1.2	7.8 ± 5.9	$.36 \pm .35$
Rime	Small	3098 ± 1559	-7.9 ± 6.4	4.6 ± 3.4	1.3 ± 1.7	9.5 ± 6.9	$.38 \pm .41$
	Light	3921 ± 1926	-11.2 ± 7.4	4.6 ± 3.8	1.6 ± 1.3	9.0 ± 6.5	$.34 \pm .34$
	Heavy	4310 ± 1632	-12.1 ± 6.3	7.1 ± 2.0	$2.2 \pm .52$	14.8 ± 6.6	N/A

Table A4. Mean and standard deviation of flight level, temperature, dew point depression ($T-T_d$), lapse rate, Showalters stability index (SSI), and liquid water content (LWC) for indicated icing type and aircraft climb rate. Temperature and dew point depression are from NGM analysis closest to time of PIREP, whereas, lapse rate, SSI, and LWC are derived from the closest upper-air sounding.

Icing Type	Climb Rate	Flight Level (m MSL)	Temperature ($^{\circ}\text{C}$)	($T-T_d$) ($^{\circ}\text{C}$)	Lapse Rate ($^{\circ}\text{C}/1000\text{ ft}$)	SSI	LWC (gm^3)
Clear	< 2000	2185 ± 1132	-3.8 ± 4.4	5.3 ± 6.3	$.91 \pm 1.9$	10.5 ± 6.9	$.31 \pm .25$
	< 3000	2813 ± 1716	-7.0 ± 6.3	4.1 ± 3.5	$.86 \pm 2.0$	11.6 ± 7.6	$.29 \pm .34$
	> 3000	3611 ± 1882	-9.8 ± 6.3	4.3 ± 2.4	1.0 ± 2.4	9.8 ± 7.4	$.15 \pm .17$
Mixed	< 2000	2300 ± 1150	-6.2 ± 4.8	4.4 ± 4.7	1.3 ± 1.4	10.3 ± 6.3	$.41 \pm .39$
	< 3000	3376 ± 1685	-9.2 ± 6.0	4.2 ± 2.9	1.1 ± 1.8	8.9 ± 6.9	$.30 \pm .28$
	> 3000	4054 ± 2060	-11.0 ± 7.7	3.9 ± 2.8	1.6 ± 1.1	8.1 ± 6.2	$.36 \pm .41$
Rime	< 2000	2625 ± 1308	-7.1 ± 5.8	4.5 ± 3.3	1.3 ± 1.9	9.6 ± 6.9	$.40 \pm .39$
	< 3000	3691 ± 1737	-9.9 ± 6.8	4.5 ± 3.4	1.5 ± 1.4	9.5 ± 6.7	$.31 \pm .36$
	> 3000	4178 ± 1961	-11.8 ± 8.0	5.0 ± 4.2	1.6 ± 1.2	8.4 ± 6.6	$.41 \pm .37$

Table A5. Mean and standard deviation of flight level, temperature, dew point depression ($T-T_d$), lapse rate, Showalters stability index (SSI), and liquid water content (LWC) for indicated icing intensity and aircraft engine type. Temperature and dew point depression are from NGM analysis closest to time of PIREP, whereas, lapse rate, SSI, and LWC are derived from the closest upper-air sounding.

Icing Intensity	Engine Type	Flight Level (m MSL)	Temperature ($^{\circ}\text{C}$)	($T-T_d$) ($^{\circ}\text{C}$)	Lapse Rate ($^{\circ}\text{C}/1000\text{ ft}$)	SSI	LWC (gm^{-3})
Trace	Piston	2596 ± 1213	-6.3 ± 5.8	4.7 ± 3.9	1.2 ± 1.9	9.9 ± 7.0	$.38 \pm .39$
	Turboprop	3442 ± 1731	-9.1 ± 6.9	4.4 ± 3.4	1.4 ± 1.3	9.7 ± 6.9	$.29 \pm .32$
	Jet	4124 ± 1953	-11.6 ± 8.1	4.8 ± 3.4	1.6 ± 1.3	8.2 ± 6.7	$.38 \pm .36$
Light	Piston	2614 ± 1283	-7.1 ± 5.0	4.3 ± 3.2	1.2 ± 1.8	9.4 ± 6.5	$.39 \pm .39$
	Turboprop	3352 ± 1644	-8.8 ± 5.6	4.3 ± 3.3	1.4 ± 1.7	9.5 ± 7.0	$.37 \pm .40$
	Jet	4276 ± 2034	-12.4 ± 7.4	4.8 ± 4.1	1.6 ± 1.4	8.9 ± 6.3	$.37 \pm .36$
Moderate	Piston	2660 ± 1231	-9.5 ± 6.3	3.6 ± 2.6	1.4 ± 1.8	10.6 ± 5.6	$.17 \pm .19$
	Turboprop	4462 ± 1499	-11.3 ± 6.0	3.6 ± 3.0	1.1 ± 2.2	8.6 ± 7.1	$.14 \pm .13$
	Jet	3880 ± 2070	-11.2 ± 7.1	4.8 ± 4.6	1.4 ± 1.2	10.1 ± 6.1	$.41 \pm .41$

Table A6. Mean and standard deviation of flight level, temperature, dew point depression ($T-T_d$), lapse rate, Showalters stability index (SSI), and liquid water content (LWC) for indicated icing type and aircraft weight class. Temperature and dew point depression are from NGM analysis closest to time of PIREP, whereas, lapse rate, SSI, and LWC are derived from the closest upper-air sounding. Data for heavy aircraft is from small sample and none of these aircraft reported moderate icing.

Icing Intensity	Weight Class	Flight Level (m MSL)	Temperature ($^{\circ}\text{C}$)	($T-T_d$) ($^{\circ}\text{C}$)	Lapse Rate ($^{\circ}\text{C}/1000\text{ ft}$)	SSI	LWC (gm^{-3})
Trace	Small	3019 ± 1553	-7.6 ± 6.7	4.6 ± 3.7	1.3 ± 1.7	9.8 ± 6.9	$.35 \pm .36$
	Light	3782 ± 1897	-10.5 ± 7.6	4.5 ± 3.5	1.5 ± 1.2	8.8 ± 6.8	$.32 \pm .34$
	Heavy	3394 ± 1380	-10.8 ± 5.9	7.4 ± 2.0	$2.2 \pm .57$	15.3 ± 6.2	N/A
Light	Small	2971 ± 1549	-7.7 ± 5.5	4.4 ± 3.4	1.3 ± 1.8	9.7 ± 7.1	$.39 \pm .42$
	Light	3914 ± 1941	-11.3 ± 7.0	4.6 ± 3.8	1.5 ± 1.4	8.9 ± 6.2	$.35 \pm .33$
	Heavy	5538 ± 1919	-15.0 ± 7.7	6.2 ± 2.3	$2.3 \pm .32$	12.8 ± 7.5	N/A
Moderate	Small	3107 ± 1618	-10.0 ± 6.5	3.6 ± 2.8	1.2 ± 2.0	10.3 ± 5.9	$.15 \pm .16$
	Light	3955 ± 1979	-11.2 ± 6.9	4.7 ± 4.4	1.5 ± 1.1	9.7 ± 6.3	$.47 \pm .42$

Table A7. Mean and standard deviation of flight level, temperature, dew point depression ($T-T_d$), lapse rate, Showalters stability index (SSI), and liquid water content (LWC) for indicated icing intensity and aircraft climb rate. Temperature and dew point depression are from NGM analysis closest to time of PIREP, whereas, lapse rate, SSI, and LWC are derived from the closest upper-air sounding.

Icing Intensity	Climb Rate	Flight Level (m MSL)	Temperature ($^{\circ}\text{C}$)	$(T-T_d)$ ($^{\circ}\text{C}$)	Lapse Rate ($^{\circ}\text{C}/1000\text{ ft}$)	SSI	LWC (gm^{-3})
Trace	< 2000	2475 ± 1217	-6.2 ± 5.6	4.6 ± 4.0	1.3 ± 1.7	9.9 ± 6.9	$.38 \pm .37$
	< 3000	3607 ± 1769	-9.5 ± 7.3	4.5 ± 3.4	1.4 ± 1.5	9.6 ± 6.9	$.29 \pm .34$
	> 3000	4003 ± 1869	-11.0 ± 8.1	4.8 ± 3.5	1.6 ± 1.1	7.9 ± 6.7	$.37 \pm .35$
Light	< 2000	2635 ± 1366	-7.1 ± 5.6	4.6 ± 3.7	1.2 ± 1.9	9.5 ± 6.6	$.42 \pm .40$
	< 3000	3540 ± 1754	-9.8 ± 5.9	4.3 ± 3.2	1.4 ± 1.5	9.4 ± 6.6	$.33 \pm .35$
	> 3000	4281 ± 2030	-12.0 ± 7.7	4.7 ± 4.2	1.5 ± 1.5	8.7 ± 6.6	$.41 \pm .41$
Moderate	< 2000	2600 ± 1302	-9.7 ± 5.6	3.5 ± 2.8	$.98 \pm 2.1$	10.9 ± 6.0	$.21 \pm .18$
	< 3000	4250 ± 1688	-10.2 ± 6.5	4.0 ± 3.5	1.3 ± 1.4	9.5 ± 6.3	$.36 \pm .46$
	> 3000	3607 ± 2116	-12.2 ± 7.6	5.2 ± 4.9	1.6 ± 1.3	9.7 ± 6.0	$.32 \pm .31$

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